Reminder (Taylor's Thm) $\sum a_n w^n$ is infinitely
for $|w| < R$, (R - radius o

$$f^{(k)}(w) = \sum_{n=k}^{\infty} \frac{n!}{(n-k)!} a_n w^{n-k}$$

$$a_n = \frac{f^{(n)}(0)}{n!}$$

Theorem Let γ be a (piecewise smooth) curve,
bounded function on γ.
For $z \notin \gamma$, let $F(z) := \oint_\gamma \frac{\varphi(\xi)}{\xi - z} d\xi$.
Then $F \in \mathcal{A}(\mathbb{C} \setminus \gamma)$.
Moreover, if $z_0 \notin \gamma$ and $R = \text{dist}(z_0, \gamma)$:
then for $|z - z_0| < R$, exists!
$F(z) = \sum a_n (z - z_0)^n$ where $a_n = \frac{F^{(n)}(z_0)}{}$

Cauchy trick: take $q = \dfrac{z - z_0}{\zeta - z_0}$, ζ

Then

$$\frac{1}{1 - \frac{z - z_0}{\zeta - z_0}} = \sum_{k=0}^{n-1} \left(\frac{z - z_0}{\zeta - z_0}\right)^k + \frac{\left(\frac{z - z_0}{\zeta - z_0}\right)^n}{1 - \frac{z - z_0}{\zeta - z_0}}$$

$$\frac{\zeta - z_0}{\zeta - z} = \sum_{k=0}^{n-1} \frac{(z - z_0)^k}{(\zeta - z_0)^k} + \frac{(z - z_0)^n}{(\zeta - z)(\zeta - z_0)^{n-}}$$

$$\boxed{\frac{1}{\zeta - z} = \sum_{k=0}^{n-1} \frac{(z - z_0)^k}{(\zeta - z_0)^{k+1}} + \frac{(z - z_0)^n}{(\zeta - z)(\zeta - z_0)^n}}$$

Multiply by $\varphi(\zeta)$, $\oint_\gamma$:

$$F(z) = \oint_\gamma \frac{\varphi(\zeta)}{\zeta - z} d\zeta = \sum_{k=0}^{n-1} (z - z_0)^k \underbrace{\oint_\gamma \frac{\varphi(\zeta}{(\zeta - }}$$

Why is F differentiable and $a_n = \dfrac{F^{(n)}(z}{n!}$

For this we show that